Calculus Formulas

The **average value** of an integrable function f on the interval [a,b] is

$$\overline{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Mean Value Theorem for Integrals

Let f be continuous on the interval [a,b]. There exists a point c in [a,b] such that

$$f(c) = \overline{f} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

Area of a Region Between Two Curves

$$A = \int_a^b (f(x) - g(x))\,dx$$

General Slicing Method

$$V = \int_a^b A(x)\,dx$$

Disk Method about the x-axis

$$V = \int_a^b \pi f(x)^2\,dx$$

Washer Method about the x-axis

$$V = \int_a^b \pi(f(x)^2 - g(x)^2)\,dx$$

Volume by the Shell Method

$$V = \int_a^b 2\pi x(f(x) - g(x))\,dx$$

Arc Length for $y = f(x)$

Let f have a continuous first derivative on the interval [a,b]

$$L = \int_a^b \sqrt{1 + f'(x)^2}\,dx$$

Mass of a one-dimensional object

On interval $a \le x \le b$ with a density function ρ (with units of mass per length)

$$m = \int_a^b \rho(x)\,dx$$

Work done by a variable force F in moving an object along a line from $x = a$ to $x = b$ in the direction of the force is

$$W = \int_a^b F(x)\,dx$$

Solving Lifting Problems

The work required to lift the water is A(y) is the cross-sectional area of the horizontal slices and D(y) is the distance the slices must be lifted.

$$W = \int_a^b \rho g A(y) D(y)\,dy$$

Solving Force/Pressure Problems

w(y) width function. If the base is at y=0 and the top is at y=a, then the total force is

$$F = \int_0^a \rho g(a - y)w(y)\,dy$$

$a - y$ depth, $w(y)$ width

Exponential Decay Functions

$$y(t) = y_0 e^{-kt}$$

Half life is $T_{1/2} = \frac{ln2}{k}$

Absolute Error $= |c - x|$

Relative Error $= \frac{|c-x|}{|x|}$

Midpoint Rule

$$M(n) = \sum_{k=1}^{n} f\left(\frac{x_{k-1}+x_k}{2}\right)\Delta x$$

Where $\Delta x = (b - a)/n$

Trapezoid Rule

$$T(n) = [\tfrac{1}{2}f(x_0) + \sum_{k=1}^{n-1} f(x_k) + \tfrac{1}{2}f(x_n)]\Delta x)$$

Where

$\Delta x = (b - a)/n \; and \; x_k = a + k\Delta x, \; for \; k = 0, 1, \ldots n)$

Simpson's Rule

$$S(n) = [f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots$$
$$+ 4(x_{n-1}) + f(x_n))\frac{\Delta x}{3}$$

$S(2n) = \frac{4T(2n)-T9n)}{3}$ when $T(2n)$ and $T(n)$ are known.

Errors in Numerical Integration

$E_M \le \frac{k(b-a)}{24}(\Delta x)^2$ and $E_T \le \frac{k(b-a)}{12}(\Delta x)^2$

$E_S \le \frac{K(b-a)}{180}(\Delta x)^4$