Business Calculus Formulas

Difference quotient (slope) over $[a, b]$

$$= \frac{f(b)-f(a)}{b-a}$$

Instantaneous Rate of Change of $f(x)$ at $x = a$ is defined as

$$f'(x) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$$

Definition of the Derivative

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

Power Rule

$$\frac{d}{dx}(x^n) = nx^{n-1}$$

Constant Rule

$$\frac{d}{dx}(cx) = c, \qquad \frac{d}{dx}(c) = 0$$

Product Rule

$$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + g'(x) \cdot f(x)$$

Quotient Rule

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - g'(x)\cdot f(x)}{[g(x)]^2}$$

Chain Rule

$$\frac{d}{dx}[f(g(x))] = f'(g(x)) \cdot g'(x)$$

Derivatives of Logs and Exponentials

$$\frac{d}{dx}(ln\, x) = \frac{1}{x}$$

$$\frac{d}{dx}(log_b x) = \frac{1}{x\, ln\, b}$$

$$\frac{d}{dx}(a^x) = a^x ln\, a$$

$$\frac{d}{dx}(e^{ax}) = ae^{ax}$$

Average and Marginal Cost

Cost function $C(x)$

Average cost $\overline{C}(x) = \frac{C(x)}{x}$

Marginal cost $C'(x)$ *(derivative)*

Stationary Points:

f has a stationary point at x if x is in the interior of the domain and $f'(x) = 0$

Singular Points:

f has a singular point at x if x is in the interior of the domain and $f'(x)$ is not defined. ($f(x)$ is defined)

Extreme Value Theorem

If f is continuous on a closed interval $[a, b]$, then it will have an absolute maximum and an absolute minimum value on that interval.

Velocity, Speed, and Acceleration

Position at time t $\quad s = f(t)$

Velocity at time t $\quad v = \frac{ds}{dt} = f'(t)$

Speed at time t $\quad |v| = |f'(t)|$

Acceleration at time t $\quad a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = f''(t)$

Price Elasticity of Demand

The percentage rate of decrease of demand per percentage increase in price.

$$E = -\frac{dq}{dp} \cdot \frac{p}{q}$$

Demand is **elastic** if $E > 1$, is **inelastic** if $E < 1$, and has **unit elasticity** if $E = 1$.

If the demand is inelastic, raising the price increases revenue.
If the demand is elastic, lowering the price increases revenue.

Basic Integration Formulas

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \qquad \int e^{ax} dx = \frac{e^{ax}}{a} + C$$

$$\int \frac{1}{x} dx = ln|x| + C \qquad \int b^{ax} dx = \frac{1}{a\, lnb} b^{ax} + C$$

$$\int |x| dx = \frac{x|x|}{2} + C$$

Left Riemann Sum over interval $[a, b]$ with n equal subdivisions

$$\sum_{k=0}^{n-1} f(x_k)\Delta x = [f(x_0) + f(x_1) + \cdots + f(x_{n-1})]\Delta x$$

Where $\Delta x = (b - a)/n$

Business Calculus Formulas

Fundamental Theorem of Calculus

$$\int_a^b f(x)dx = F(b) - F(a)$$

Integration by Parts

$$\int u \cdot v = u \cdot v - \int u' \cdot v\, dx$$

Area of a Region Between Two Curves

If $f(x) > g(x)$ for all x in $[a, b]$

$$A = \int_a^b (f(x) - g(x))\, dx$$

Average Value of a Function (mean)

$$\overline{f} = \frac{1}{b-a}\int_a^b f(x)\, dx$$

Consumers' Surplus

If demand for an item is given by $p = D(q)$, the selling price is $\overline{p}$, and $\overline{q}$ is the corresponding demand, then the consumers' surplus is the difference between willingness to spend and actual expenditure.

$$CS = \int_0^{\overline{q}} D(q)dq = \overline{p}\,\overline{q} = \int_0^{\overline{q}} (D(q) - \overline{p})dq$$

Producers' Surplus

The extra amount earned by producers who were willing to charge less than the selling price of $\overline{p}$ per unit.

$$PS = \int_0^{\overline{q}} [\overline{p} - S(q)]dq, \text{ where } S(\overline{q}) = \overline{p}.$$

Social Gain = $CS + PS$

Total Value of a Continuous Income Stream

$$TV = \int_a^b R(t)dt$$

Future Value of a Continuous Income Stream

$$FV = \int_a^b R(t)e^{r(b-t)}dt$$

Present Value of a Continuous Income Stream

$$PV = \int_a^b R(t)e^{r(a-t)}dt$$

Newton's Law of Cooling, where T_s is the ambient surrounding temperature.

$$\frac{dT}{dt} =- k(T - Ts)$$